Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Finding an Indefinite Integral **In Exercises 1–8, find the indefinite integral.**

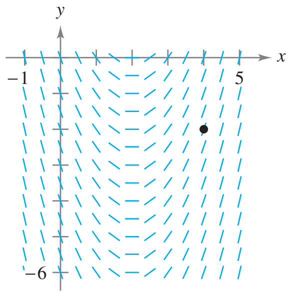
1. $\int (x - 6)\, dx$
2. $\int (x^4 + 3)\, dx$
3. $\int (4x^2 + x + 3)\, dx$
4. $\int \frac{6}{\sqrt[3]{x}}\, dx$
5. $\int \frac{x^4 + 8}{x^3}\, dx$
6. $\int \frac{x^2 + 2x - 6}{x^4}\, dx$
7. $\int (2x - 9 \sin x)\, dx$
8. $\int (5 \cos x - 2 \sec^2 x)\, dx$

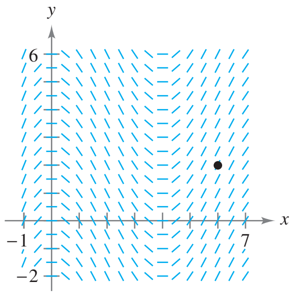
Finding a Particular Solution **In Exercises 9–12, find the particular solution that satisfies the differential equation and the initial condition.**

9. $f'(x) = -6x,\ f(1) = -2$
10. $f'(x) = 9x^2 + 1,\ f(0) = 7$
11. $f''(x) = 24x,\ f'(-1) = 7,\ f(1) = -4$
12. $f''(x) = 2 \cos x,\ f'(0) = 4,\ f(0) = -5$

Slope Field **In Exercises 13 and 14, a differential equation, a point, and a slope field are given. (a) Sketch two approximate solutions of the differential equation on the slope field, one of which passes through the indicated point. (To print an enlarged copy of the graph, go to *MathGraphs.com*.) (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution.**

13. $\frac{dy}{dx} = 2x - 4, \quad (4, -2)$
14. $\frac{dy}{dx} = \frac{1}{2}x^2 - 2x, \quad (6, 2)$





15. **Velocity and Acceleration** A ball is thrown vertically upward from ground level with an initial velocity of 96 feet per second. Use $a(t) = -32$ feet per second per second as the acceleration due to gravity. (Neglect air resistance.)
    (a) How long will it take the ball to rise to its maximum height? What is the maximum height?
    (b) After how many seconds is the velocity of the ball one-half the initial velocity?
    (c) What is the height of the ball when its velocity is one-half the initial velocity?

16. **Velocity and Acceleration** The speed of a car traveling in a straight line is reduced from 45 to 30 miles per hour in a distance of 264 feet. Find the distance in which the car can be brought to rest from 30 miles per hour, assuming the same constant deceleration.

17. **Velocity and Acceleration** An airplane taking off from a runway travels 3600 feet before lifting off. The airplane starts from rest, moves with constant acceleration, and makes the run in 30 seconds. With what speed does it lift off?

18. **Modeling Data** The table shows the velocities (in miles per hour) of two cars on an entrance ramp to an interstate highway. The time t is in seconds.

t	v_1	v_2
0	0	0
5	2.5	21
10	7	38
15	16	51
20	29	60
25	45	64
30	65	65

    (a) Rewrite the velocities in feet per second.
    (b) Use the regression capabilities of a graphing utility to find quadratic models for the data in part (a).
    (c) Approximate the distance traveled by each car during the 30 seconds. Explain the difference in the distances.

Finding a Sum **In Exercises 19 and 20, find the sum. Use the summation capabilities of a graphing utility to verify your result.**

19. $\sum_{i=1}^{5} (5i - 3)$
20. $\sum_{k=0}^{3} (k^2 + 1)$

Using Sigma Notation **In Exercises 21 and 22, use sigma notation to write the sum.**

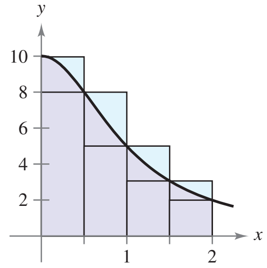
21. $\frac{1}{3(1)} + \frac{1}{3(2)} + \frac{1}{3(3)} + \cdots + \frac{1}{3(10)}$
22. $\left(\frac{3}{n}\right)\left(\frac{1+1}{n}\right)^2 + \left(\frac{3}{n}\right)\left(\frac{2+1}{n}\right)^2 + \cdots + \left(\frac{3}{n}\right)\left(\frac{n+1}{n}\right)^2$

Evaluating a Sum **In Exercises 23–28, use the properties of summation and Theorem 4.2 to evaluate the sum.**

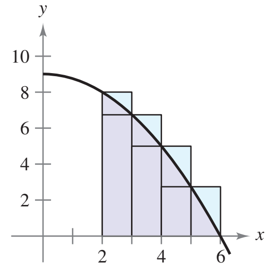
23. $\sum_{i=1}^{24} 8$
24. $\sum_{i=1}^{75} 5i$
25. $\sum_{i=1}^{20} 2i$
26. $\sum_{i=1}^{30} (3i - 4)$
27. $\sum_{i=1}^{20} (i + 1)^2$
28. $\sum_{i=1}^{12} i(i^2 - 1)$

Finding Upper and Lower Sums for a Region **In Exercises 29 and 30, use upper and lower sums to approximate the area of the region using the given number of subintervals (of equal width.)**

29. $y = \dfrac{10}{x^2 + 1}$



30. $y = 9 - \dfrac{1}{4}x^2$



Finding Area by the Limit Definition **In Exercises 31–34, use the limit process to find the area of the region bounded by the graph of the function and the x-axis over the given interval. Sketch the region.**

31. $y = 8 - 2x, \quad [0, 3]$

32. $y = x^2 + 3, \quad [0, 2]$

33. $y = 5 - x^2, \quad [-2, 1]$

34. $y = \frac{1}{4}x^3, \quad [2, 4]$

35. Finding Area by the Limit Definition Use the limit process to find the area of the region bounded by $x = 5y - y^2$, $x = 0$, $y = 2$, and $y = 5$.

36. Upper and Lower Sums Consider the region bounded by $y = mx$, $y = 0$, $x = 0$, and $x = b$.

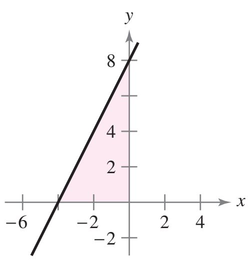
(a) Find the upper and lower sums to approximate the area of the region when $\Delta x = b/4$.

(b) Find the upper and lower sums to approximate the area of the region when $\Delta x = b/n$.

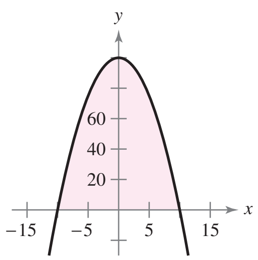
(c) Find the area of the region by letting n approach infinity in both sums in part (b). Show that, in each case, you obtain the formula for the area of a triangle.

Writing a Definite Integral **In Exercises 37 and 38, set up a definite integral that yields the area of the region. (Do not evaluate the integral.)**

37. $f(x) = 2x + 8$



38. $f(x) = 100 - x^2$



Evaluating a Definite Integral Using a Geometric Formula **In Exercises 39 and 40, sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral.**

39. $\displaystyle\int_0^5 (5 - |x - 5|)\, dx$

40. $\displaystyle\int_{-6}^6 \sqrt{36 - x^2}\, dx$

41. Using Properties of Definite Integrals Given

$$\int_4^8 f(x)\, dx = 12 \quad \text{and} \quad \int_4^8 g(x)\, dx = 5$$

evaluate

(a) $\displaystyle\int_4^8 [f(x) + g(x)]\, dx.$ (b) $\displaystyle\int_4^8 [f(x) - g(x)]\, dx.$

(c) $\displaystyle\int_4^8 [2f(x) - 3g(x)]\, dx.$ (d) $\displaystyle\int_4^8 7f(x)\, dx.$

42. Using Properties of Definite Integrals Given

$$\int_0^3 f(x)\, dx = 4 \quad \text{and} \quad \int_3^6 f(x)\, dx = -1$$

evaluate

(a) $\displaystyle\int_0^6 f(x)\, dx.$ (b) $\displaystyle\int_6^3 f(x)\, dx.$

(c) $\displaystyle\int_4^4 f(x)\, dx.$ (d) $\displaystyle\int_3^6 -10f(x)\, dx.$

Evaluating a Definite Integral **In Exercises 43–50, use the Fundamental Theorem of Calculus to evaluate the definite integral.**

43. $\displaystyle\int_0^8 (3 + x)\, dx$

44. $\displaystyle\int_2^3 (t^2 - 1)\, dt$

45. $\displaystyle\int_{-1}^1 (4t^3 - 2t)\, dt$

46. $\displaystyle\int_2^3 (x^4 + 4x - 6)\, dx$

47. $\displaystyle\int_4^9 x\sqrt{x}\, dx$

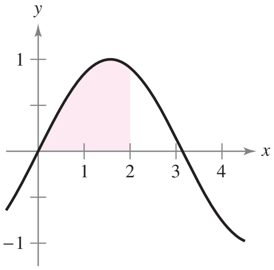
48. $\displaystyle\int_1^4 \left(\frac{1}{x^3} + x\right) dx$

49. $\displaystyle\int_0^{3\pi/4} \sin\theta\, d\theta$

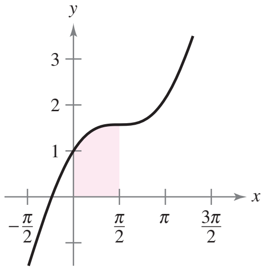
50. $\displaystyle\int_{-\pi/4}^{\pi/4} \sec^2 t\, dt$

Finding the Area of a Region **In Exercises 51 and 52, determine the area of the given region.**

51. $y = \sin x$



52. $y = x + \cos x$



Finding the Area of a Region **In Exercises 53–56, find the area of the region bounded by the graphs of the equations.**

53. $y = 8 - x, x = 0, x = 6, y = 0$

54. $y = -x^2 + x + 6, y = 0$

55. $y = x - x^3, x = 0, x = 1, y = 0$

56. $y = \sqrt{x}(1 - x), y = 0$

Finding the Average Value of a Function **In Exercises 57 and 58, find the average value of the function over the given interval and all values of x in the interval for which the function equals its average value.**

57. $f(x) = \dfrac{1}{\sqrt{x}},\quad [4, 9]$ **58.** $f(x) = x^3,\quad [0, 2]$

Using the Second Fundamental Theorem of Calculus **In Exercises 59–62, use the Second Fundamental Theorem of Calculus to find $F'(x)$.**

59. $F(x) = \displaystyle\int_0^x t^2\sqrt{1 + t^3}\, dt$ **60.** $F(x) = \displaystyle\int_1^x \frac{1}{t^2}\, dt$

61. $F(x) = \displaystyle\int_{-3}^x (t^2 + 3t + 2)\, dt$

62. $F(x) = \displaystyle\int_0^x \csc^2 t\, dt$

Finding an Indefinite Integral **In Exercises 63–72, find the indefinite integral.**

63. $\displaystyle\int \frac{x^2}{\sqrt{x^3 + 3}}\, dx$ **64.** $\displaystyle\int 6x^3\sqrt{3x^4 + 2}\, dx$

65. $\displaystyle\int x(1 - 3x^2)^4\, dx$ **66.** $\displaystyle\int \frac{x + 4}{(x^2 + 8x - 7)^2}\, dx$

67. $\displaystyle\int \sin^3 x \cos x\, dx$ **68.** $\displaystyle\int x \sin 3x^2\, dx$

69. $\displaystyle\int \frac{\cos\theta}{\sqrt{1 - \sin\theta}}\, d\theta$

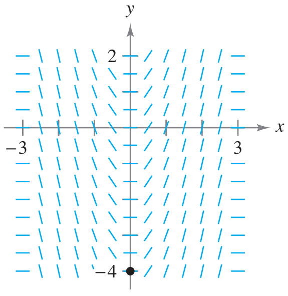
70. $\displaystyle\int \frac{\sin x}{\sqrt{\cos x}}\, dx$

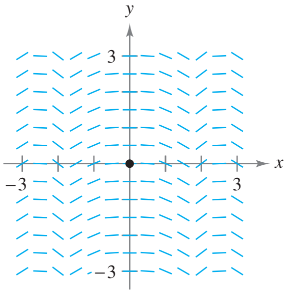
71. $\displaystyle\int (1 + \sec \pi x)^2 \sec \pi x \tan \pi x\, dx$

72. $\displaystyle\int \sec 2x \tan 2x\, dx$

Slope Field **In Exercises 73 and 74, a differential equation, a point, and a slope field are given. (a) Sketch two approximate solutions of the differential equation on the slope field, one of which passes through the given point. (To print an enlarged copy of the graph, go to *MathGraphs.com*.) (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution.**

73. $\dfrac{dy}{dx} = x\sqrt{9 - x^2},\quad (0, -4)$ **74.** $\dfrac{dy}{dx} = -\dfrac{1}{2}x\sin(x^2),\quad (0, 0)$





Evaluating a Definite Integral **In Exercises 75–82, evaluate the definite integral. Use a graphing utility to verify your result.**

75. $\displaystyle\int_0^1 (3x + 1)^5\, dx$ **76.** $\displaystyle\int_0^1 x^2(x^3 - 2)^3\, dx$

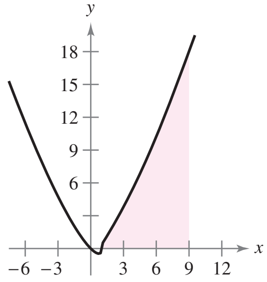
77. $\displaystyle\int_0^3 \frac{1}{\sqrt{1 + x}}\, dx$ **78.** $\displaystyle\int_3^6 \frac{x}{3\sqrt{x^2 - 8}}\, dx$

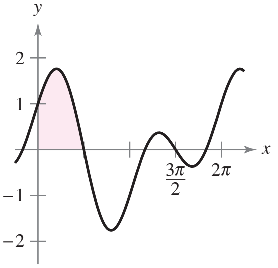
79. $2\pi\displaystyle\int_0^1 (y + 1)\sqrt{1 - y}\, dy$ **80.** $2\pi\displaystyle\int_{-1}^0 x^2\sqrt{x + 1}\, dx$

81. $\displaystyle\int_0^{\pi} \cos\frac{x}{2}\, dx$ **82.** $\displaystyle\int_{-\pi/4}^{\pi/4} \sin 2x\, dx$

Finding the Area of a Region **In Exercises 83 and 84, find the area of the region. Use a graphing utility to verify your result.**

83. $\displaystyle\int_1^9 x\sqrt[3]{x - 1}\, dx$ **84.** $\displaystyle\int_0^{\pi/2} [\cos x + \sin(2x)]\, dx$





85. Using an Even Function Use $\displaystyle\int_0^2 x^4\, dx = \frac{32}{5}$ to evaluate each definite integral without using the Fundamental Theorem of Calculus.

(a) $\displaystyle\int_{-2}^2 x^4\, dx$ (b) $\displaystyle\int_{-2}^0 x^4\, dx$

(c) $\displaystyle\int_0^2 3x^4\, dx$ (d) $\displaystyle\int_{-2}^0 -5x^4\, dx$

86. Respiratory Cycle After exercising for a few minutes, a person has a respiratory cycle for which the rate of air intake is

$$v = 1.75 \sin\frac{\pi t}{2}.$$

Find the volume, in liters, of air inhaled during one cycle by integrating the function over the interval $[0, 2]$.

Using the Trapezoidal Rule and Simpson's Rule **In Exercises 87–90, approximate the definite integral using the Trapezoidal Rule and Simpson's Rule with $n = 4$. Compare these results with the approximation of the integral using a graphing utility.**

87. $\displaystyle\int_2^3 \frac{2}{1 + x^2}\, dx$ **88.** $\displaystyle\int_0^1 \frac{x^{3/2}}{3 - x^2}\, dx$

89. $\displaystyle\int_0^{\pi/2} \sqrt{x}\cos x\, dx$ **90.** $\displaystyle\int_0^{\pi} \sqrt{1 + \sin^2 x}\, dx$